Recall that the logarithmic and exponential functions “undo” each other. This means that logarithms have similar properties to exponents. Some important properties of logarithms are given here.

# Using the Product Rule for Logarithms

Recall that we use the product rule of exponents to combine the product of exponents by adding: . We have a similar property for logarithms, called the **product rule for logarithms**, which says that the logarithm of a product is equal to a sum of logarithms.

The **product rule for logarithms** can be used to simplify a logarithm of a product by rewriting it as a sum of individual logarithms.

for

and where and are positive real numbers.

We can use the product rule to rewrite logarithms by

Factoring the argument completely, expressing each whole number factor as a product of primes.

Writing the equivalent expression by summing the logarithms of each factor.

Examples: Use the product rule to rewrite the following logarithms.

# Using the Quotient Rule for Logarithms

Recall that we use the quotient rule of exponentsto combine the quotient of exponents by subtracting:  . The **quotient rule for logarithms** says that the logarithm of a quotient is equal to a difference of logarithms.

The **quotient rule for logarithms** can be used to simplify a logarithm or a quotient by rewriting it as the difference of individual logarithms.

where and are positive real numbers.

We can use the quotient rule to rewrite logarithms by

Express the argument in lowest terms by factoring the numerator and denominator and canceling common terms.

Write the equivalent expression by subtracting the logarithm of the denominator from the logarithm of the numerator.

Check to see that each term is fully expanded. If not, apply the product rule for logarithms to expand completely.

Examples: Use the quotient rule to rewrite the following logarithms.

# Using the Power Rule for Logarithms

The **power rule for logarithms** can be used to simplify the logarithm of a power by rewriting it as the product of the exponent times the logarithm of the base.

where is a positive real number.

We can use the power rule to rewrite logarithms by

Express the argument as a power, if needed.

Write the equivalent expression by multiplying the exponent times the logarithm of the base.

Examples: Rewrite the following logarithms using the power rule.

Taken together, the product rule, quotient rule, and power rule are often called “laws of logs.” We often apply more than one rule in order to simplify an expression.

# Expanding Logarithmic Functions

When asked to **expand a logarithmic function**, it typically means one log expression is given with a complicated argument and we use the laws of logs to separate the log apart into many log terms, each with only one simple argument.

Examples: Expand the following logarithms using the laws of logs.

# Condensing Logarithmic Expressions

When asked to **condense a logarithmic function,** it typically means you are given a string of log expressions, each with a simple argument, and will need to “work backwards” to create one log expression with a complicated argument. It is important to remember that the logarithms must have the same base to be combined.

Examples: Condense the following logarithms as a single logarithm using the law of logs.

# Using the Change-of-Base Formula for Logarithms

Some calculators can evaluate only common and natural logs. In order to evaluate logarithms with a base other than 10 or , we use the change-of-base formula to rewrite the logarithm as the quotient of logarithms of any other base; when using a calculator, we would change them to common or natural logs.

The change-of-base formula can be used to evaluate a logarithm with any base. For any positive real numbers , , and , where and ,

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It follows that the change-of-base formula can be used to rewrite a logarithm with any base as the quotient of common or natural logs.

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and

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Examples:

1. Change to a quotient of natural logarithms.
2. Evaluate using the change-of-base formula with a calculator.